ALTAYEVA AIZHAN BAKATKALIEVNA

VARIANTS OF BINARITY AND COUNTABLE CATEGORICITY FOR ORDERED STRUCTURES

ABSTRACT

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Actuality of the research theme. The dissertation work is related to one of the important sections of mathematical logic - the theory of models, which studies the relationship between a formal language and its interpretations, or models. One of the classical objects of study in the theory of first-order models are complete theories, which in turn are divided into stable and non stable theories. Since the end of the sixties of the twentieth century, the theory of models has mainly developed in the direction of studying the properties of stable theories, many deep theorems on the structure of their models have been obtained. At the same time, the most interesting and fruitful ideas and methods, such as the theory of dimensions, orthogonality, modularity , and depth of types, arose in the study of subclasses of stable theories with natural restrictions on formula sets. In particular, the concept of strong minimality, introduced by A. Lachlan and J. Baldwin , had a significant stimulating effect on the creation of the above methods.

At the same time, for a long time, the class of linearly ordered structures, a subclass of the class of unstable theories, has been an important subject of study for specialists in model theory. During this time, in the study of models of some particular theories that extend the theory of linear order, impressive results were obtained. Among the theories to which successful approaches have been found are Peano arithmetic, the theory of ordered Abelian groups, the theory of real closed fields, and the theory of linear order itself. However, in model theory, the general theory of ordered structures began to develop only after Anand Pillai and Charles Steinhorn introduced the concept of o-minimality in a series of joint papers in 1984, which turned out to be very fruitful (Note that the introduction of the concept of o-minimality was directly inspired by the paper L. van den Dries , where he studied o-minimal enrichments $\langle R, < \rangle$). Recall that a linearly ordered structure is called o-minimal if any of its formula subsets is a finite union of intervals and points. A classic example of an o-minimal structure is an ordered field of real numbers. Today, o-minimality is a very developed area in model theory with numerous applications not only in mathematical logic, but also in analytic and differential geometry, and function theory. Here one cannot fail to note the outstanding result of Alex Wilka, who is directly related to the theory of functions and geometry and states that the complement of a finite number of projections of polynomial-exponential equalities and inequalities is itself a projection of polynomial-exponential equalities and inequalities, in other words, that the

enrichment of an ordered field of real numbers exponential has model complete and o-minimal theory. The relevance of the topic of the dissertation work is explained by the fact that one of the main directions of modern model theory is the study of o-minimality and its various variants.

In this paper, the applicant continues the study of the concept of weak o-minimality. Particular emphasis is placed on the study of the relationship between the concepts of binary and almost omega- categorical. Along with this, we study the notion of weak circular minimality, which is a variant of o-minimality for circularly ordered sets. Any linearly ordered structure is a $\varnothing$-definable circularly ordered structure, and for a given circularly ordered structure, there exists a definable linear order over any parameter. There is a very close connection between weak circular minimality and weak o-minimality. At the same time, there are differences that arise between these notions, which include definability over $\varnothing$, thus determining an independent interest in the study of weak circular minimality. Here we consider the relationship between the concepts of almost binary and countable categoricity.

The aims and objectives of the study.

The work is devoted to the study of the relationship between the concepts of binary and almost omega-categoricity in weakly o-minimal theories, as well as the study of the relationship between the concepts of almost binary and countable categoricity in weakly circularly minimal theories.

Research objectives:

1. Investigate general structural properties of variants of o-minimality: weak o-minimality and weak circular minimality.
2. Investigate the properties of almost omega-categorical weakly o-minimal theories.
3. Investigate the properties of countably categorical weakly circularly minimal theories.

The main provisions for the defence of the dissertation:

1. Orthogonality of an arbitrary finite family of non-algebraic pairwise weakly orthogonal 1-types in almost omega-categorical completely o-minimal theories.
2. Binarity of almost omega-categorical completely o-minimal theories.
3. Almost omega-categoricity of weakly o-minimal theories of finite convexity rank having a small number of countable models.
4. Description of the algebra of binary isolating formulas over a non-algebraic 1-type over an empty set for almost omega-categorical weakly o-minimal theories.
5. A criterion for the generalized commutativity of the algebra of binary isolating formulas over a pair of not weakly orthogonal non-algebraic 1-types.
6. Description of countably categorical m-convex weakly circularly minimal theories of convexity rank 1 that are almost binary ($m > 1$).
7. A criterion for the almost binary nature of countably categorical non-1-transitive weakly circularly minimal structures in terms of convexity rank.

The objects of research are linearly ordered structures with the weak o-minimality property; circularly ordered structures with the weak circular minimality property.

Research methods include classical methods of model theory. In addition to these methods, we used the methodology of studying ordered structures based on such concepts as o-minimality and variants of o-minimality. Typical in such a situation is the imposition of strict restrictions on the sets defined by the formula with one free variable. Thus, an o-minimal structure M can be regarded as an L-structure, where $L \supset L_0 = \{<\}$, $<$ is a linear order on M, and every definable subset of the structure M is quantifier-free L_0-definable. This gives a setting for other concepts: we replace it L_0 with some other well-known language, we consider L-structures such that L_0- the reduct has a conditional type (for example, a linear order), and we require that every definable subset of the structure M is (quantifier-free) L_0-definable (one can require this for all models of the given theory). Some of the necessary tools are created in the course of writing the work.

Scientific novelty of dissertation research. New scientific results are obtained in the study of both weakly o-minimal theories and weakly circularly minimal theories.

Theoretical and practical significance of the research. The work is theoretical. The results obtained can be used in model theory, group theory, field theory. Some of the results obtained have been applied to the study of database queries over an ordered domain; in particular, the issues of reducibility of extended queries to restricted ones were considered .

Connection of the dissertation with other research works. The dissertation was implemented within the framework of a scientific project of grant funding by the CS MES RK in the priority direction of science development "Scientific research in the field of natural sciences", a specialized scientific direction "Fundamental and applied research in the field of mathematics and mechanics" on the topic "Structural properties of almost omega-categorical ordered theories" (2020-2022, AP0 8855544).

The work approbation. The results of the dissertation were presented at the Logical Colloquium (Poznan, Poland, 2021), at the International Conferences on Mathematical Logic "Maltsev Readings" dedicated to the memory of A.I. Maltsev (Novosibirsk, 2020, 2021), at the International Scientific Conference "Actual Problems of Analysis, Differential Equations and Algebra" (Nur -Sultan, 2019), at the Traditional International April Mathematical Conference IMMM (Almaty, 2020, 2021), at the International Conference " Mathematical modeling and information technologies in education and science" (Almaty, 2020).

Assessment of the completeness of the aims of the work. All tasks of the dissertation research are fully completed. The reliability and validity of the results obtained is confirmed by publication in peer-reviewed scientific journals, speeches at various scientific conferences.

Evaluation of the scientific level of work in comparison with achievements in the scientific direction. The results obtained are new, published in peer-reviewed scientific journals indexed by the Web . of Science and (or) Scopus , and make a significant contribution to the study of o-minimality options.

Publications. The main results of the dissertation were published in 15 papers, including 4 articles in journals with a non-zero impact factor in accordance with international databases Web of Science and/or Scopus : Siberian Mathematical Journal , Siberian Electronic Mathematical Reports , Mathematical Notes , Algebra and logic ; 2 articles in journals "Bulletin of KazNU , a series of mathematics, mechanics, computer science" and "Bulletin of KBTU" (recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan); and 9 works in the proceedings of international conferences.

The volume and structure of the dissertation. The dissertation consists of an introduction, six sections, a conclusion, a list of references. The total volume of the dissertation is 103 pages, the list of references includes 71 papers.

The main content of the dissertation. The introduction includes the relevance of the research topic; work goals; the main provisions submitted for defense; objects of study; research methods; scientific novelty; theoretical and practical significance of the study; approbation of work.

The first section is devoted to the study of the almost omega-categoricity of weakly o-minimal theories of finite convexity rank that have a small number of countable models.

The second section is devoted to the question of the binarity of almost omega-categorical completely o-minimal theories.

The third section is devoted to the description of distribution algebras of binary isolating formulas for almost omega-categorical weakly o-minimal theories.

The fourth section is devoted to a complete description of countably categorical n -convex weakly circularly minimal theories of convexity rank 1 that are almost binary.

The fifth section is devoted to finding necessary and sufficient conditions for the almost binary nature of countable categorical weakly circularly minimal theories.

The sixth section is devoted to studying the properties of E-combinations of countable categorical linear orders.

In conclusion, the main results of the dissertation work are presented and summarized.